ME 141

Engineering Mechanics

Lecture 4: Equilibrium of rigid bodies

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Courtesy: Vector Mechanics for Engineers, Beer and Johnston

Introduction

- For a rigid body in static equilibrium, the external forces and moments are balanced and will impart no translational or rotational motion to the body.

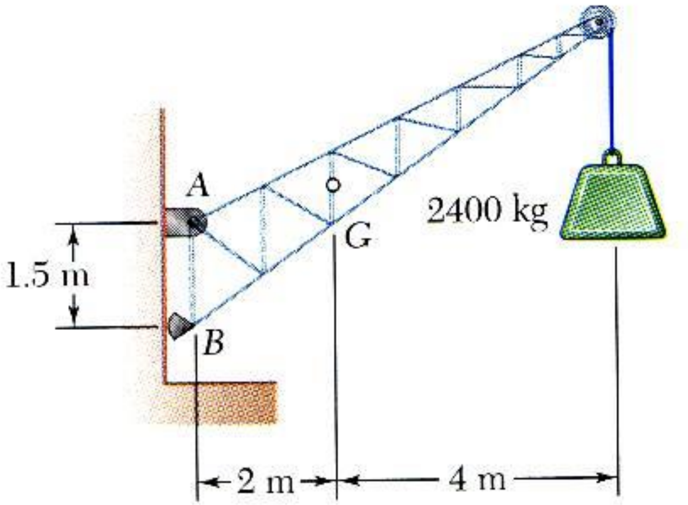
- The necessary and sufficient condition for the static equilibrium of a body are that **the resultant force and couple from all external forces form a system equivalent to zero,**

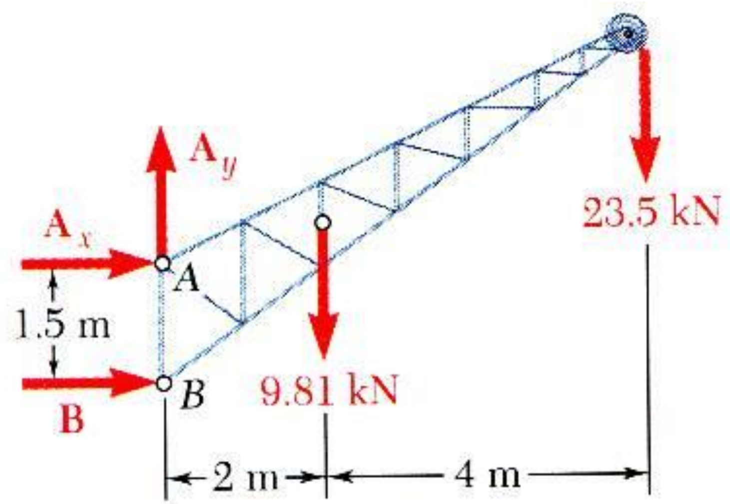
$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$

- Resolving each force and moment into its rectangular components leads to 6 scalar equations which also express the conditions for static equilibrium,

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

Free-Body Diagram

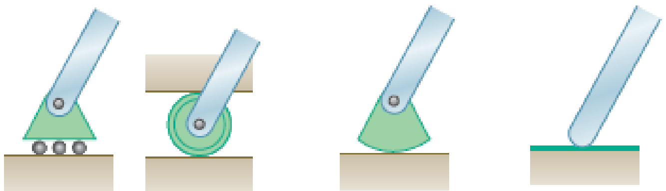
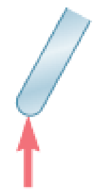
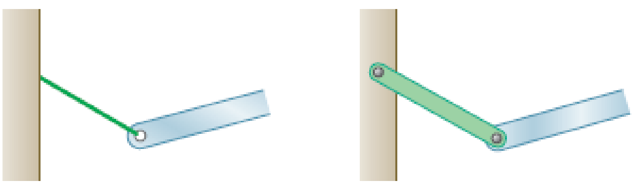
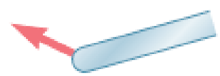
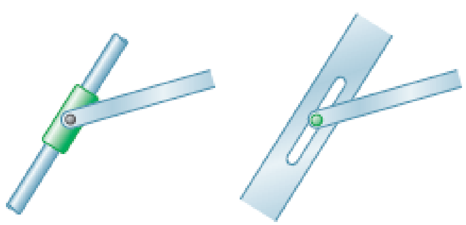
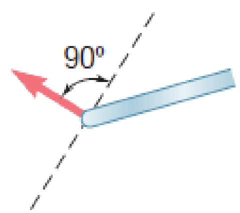




First step in the static equilibrium analysis of a rigid body is identification of all forces acting on the body with a *free-body* diagram.

- Select the extent of the free-body and detach it from the ground and all other bodies.
- Indicate point of application, magnitude, and direction of external forces, including the rigid body weight.
- Indicate point of application and assumed direction of unknown applied forces. These usually consist of reactions through which the ground and other bodies oppose the possible motion of the rigid body.
- Include the dimensions necessary to compute the moments of the forces.

Reactions at Supports and Connections for a Two-Dimensional Structure

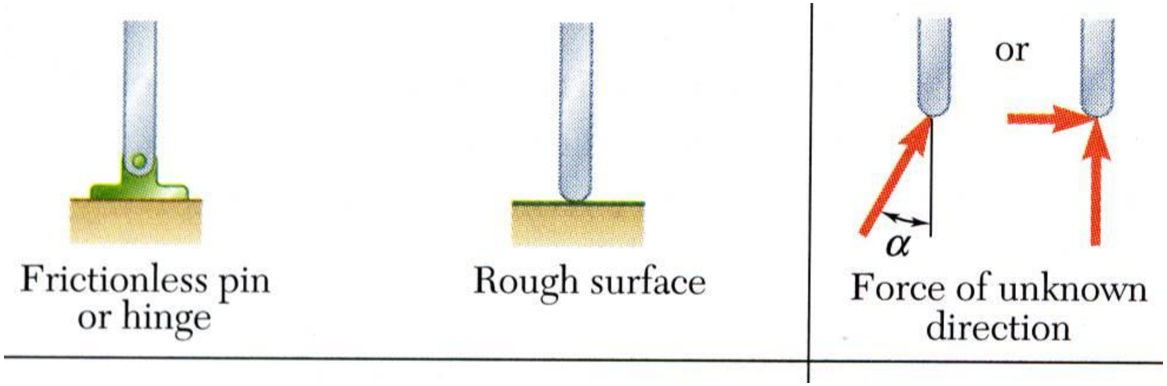
Support or Connection	Reaction	Number of Unknowns
 Rollers, Rocker, Frictionless surface	 Force with known line of action	1
 Short cable, Short link	 Force with known line of action	1
 Collar on frictionless rod, Frictionless pin in slot	 Force with known line of action	1

1. **Reactions equivalent to a force with known line of action:**

Rollers, rockers, frictionless surfaces, short links and cables, collars on frictionless rods, and frictionless pins in slots

- Each of these supports and connections can prevent motion in one direction only

Reactions at Supports and Connections for a Two-Dimensional Structure

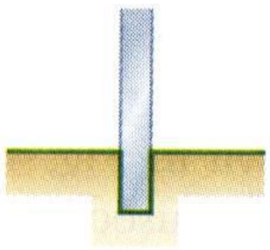


Number of Unknowns
2

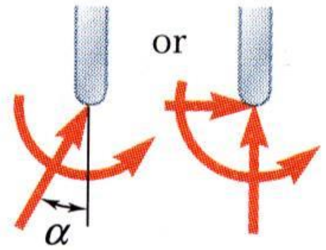
2. Reactions Equivalent to a Force of Unknown Direction and Magnitude: frictionless pins in fitted holes, hinges, and rough surfaces

- They can prevent translation of the free body in all directions, but they cannot prevent the body from rotating about the connection
- Reactions of this group involve two unknowns and are usually represented by their x and y components

Reactions at Supports and Connections for a Two-Dimensional Structure



Fixed support



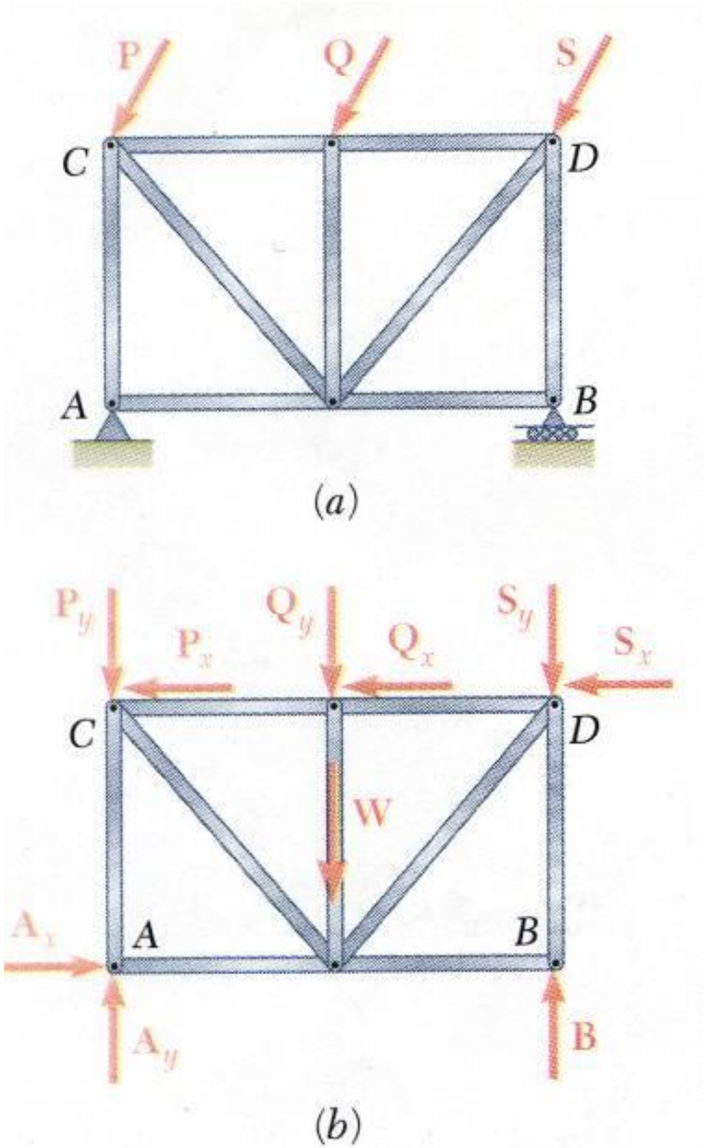
Force and couple

Number of Unknowns
3

3. **Reactions Equivalent to a Force and a Couple:** Fixed supports, which oppose any motion of the free body and thus constrain it completely

- Fixed supports actually produce forces over the entire surface of contact
- These forces, however, form a system which can be reduced to a force and a couple.
- Reactions of this group involve three unknowns, consisting usually of the two components of the force and the moment of the couple.

Equilibrium of a Rigid Body in Two Dimensions



- For all forces and moments acting on a two-dimensional structure,

  $$F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$$

- Equations of equilibrium become

  $$\boxed{\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0}$$

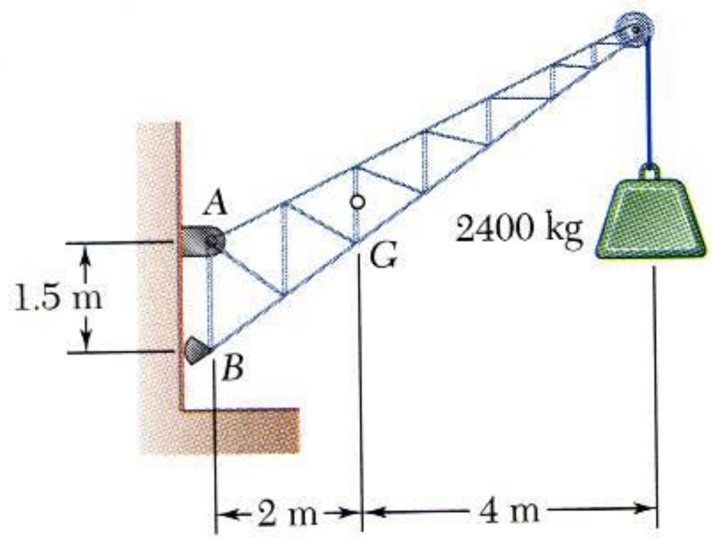
  where A is any point in the plane of the structure.

- The 3 equations can be solved for no more than 3 unknowns.

- The 3 equations can not be augmented with additional equations, but they can be replaced

  $$\boxed{\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0}$$

Problem 4.1

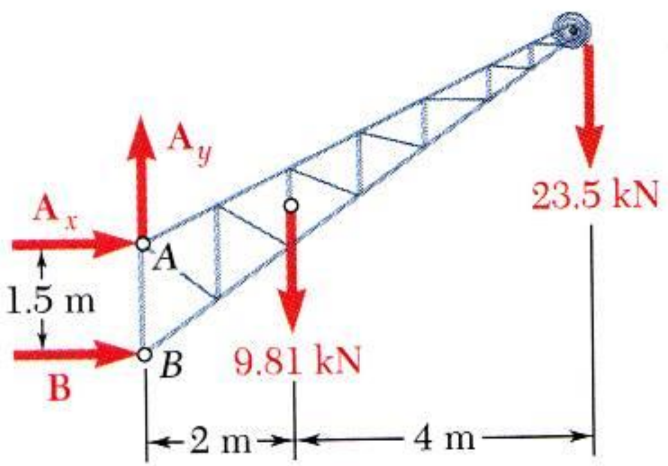


A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*.

Determine the components of the reactions at *A* and *B*.

Problem 4.1

Free-body diagram of the structure,



- Determine B by solving the equation for the sum of the moments of all forces about A.

$$\sum M_A = 0: \quad +B(1.5\text{m}) - 9.81\,\text{kN}(2\text{m}) - 23.5\,\text{kN}(6\text{m}) = 0$$

$$\boxed{B = +107.1\,\text{kN}}$$

- Determine the reactions at A by solving the equations for the sum of all horizontal forces and all vertical forces.

$$\sum F_x = 0: \quad A_x + B = 0$$

$$\boxed{A_x = -107.1\,\text{kN}}$$

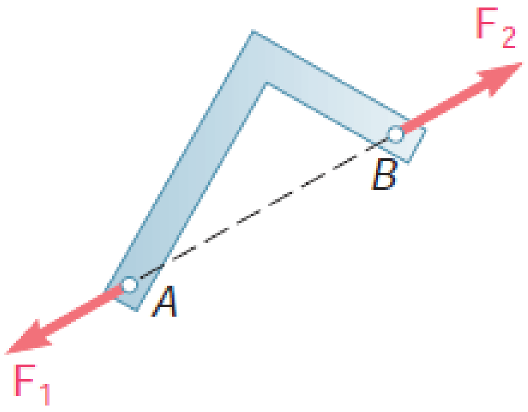
$$\sum F_y = 0: \quad A_y - 9.81\,\text{kN} - 23.5\,\text{kN} = 0$$

$$\boxed{A_y = +33.3\,\text{kN}}$$

Equilibrium of a Two-Force Body

A particular case of equilibrium which is of considerable interest is that of a rigid body subjected to two forces. Such a body is commonly called a *two-force body*

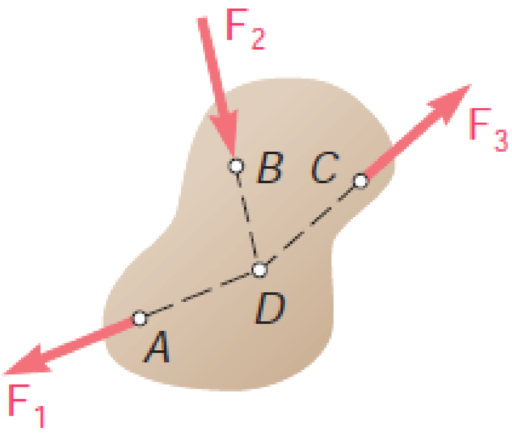
If a two-force body is in equilibrium, the two forces must have the same magnitude, the same line of action, and opposite sense.



Equilibrium of a Three-Force Body

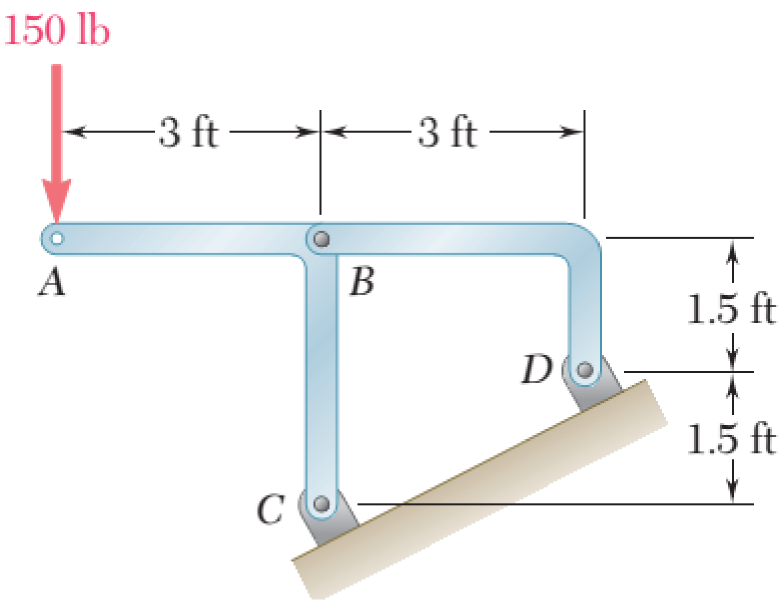
Another case of equilibrium that is of great interest is that of a *three force body,* i.e., a rigid body subjected to three forces or, more generally, *a rigid body subjected to forces acting at only three points***.**

If the body is in equilibrium, *the lines of action of the three forces must be either concurrent or parallel.*



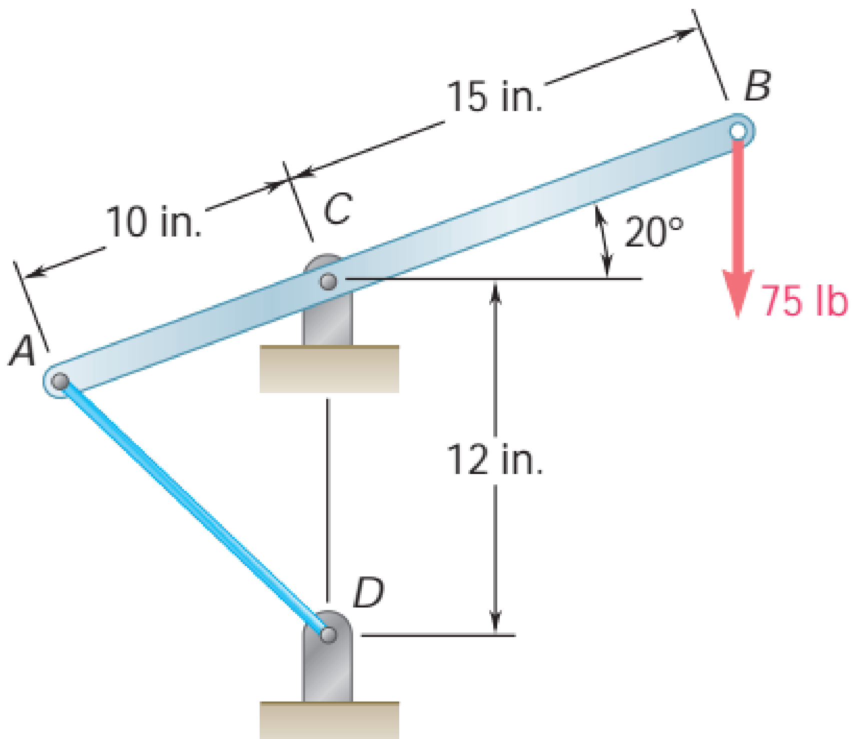
Prob 4.66

For the frame and loading shown, determine the reactions at C and D.



Prob # 4.24

A lever *AB* is hinged at *C* and attached to a control cable at *A*. If the lever is subjected to a 75-lb vertical force at *B*, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.



Equilibrium of a Rigid Body in Three Dimensions

- Six scalar equations are required to express the conditions for the equilibrium of a rigid body in the general three dimensional case.

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

- These equations can be solved for no more than 6 unknowns which generally represent reactions at supports or connections.

- The scalar equations are conveniently obtained by applying the vector forms of the conditions for equilibrium,

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$